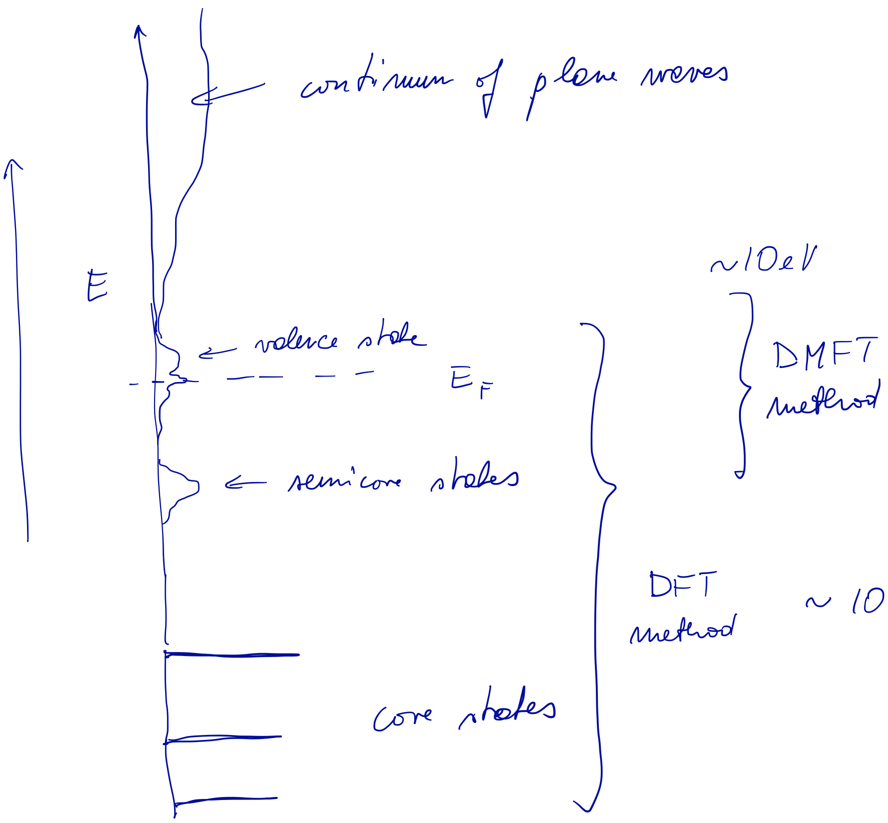
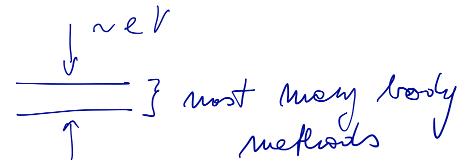
Quasiparticles (Richard Martin Chapter 7)

Origin of the frequency dependence of self energy.

We almost never treat all degrees of freedom in a solid, but almost always divide it into the "relevant" low energy degrees of freedom and the rest.

Exact theory would need to address extremely large energy window $\sim 10^4$ eV (10^8 K)





In practice it is convenient to write H in block form:

$$\begin{bmatrix} H_S , & H_{SR} \\ H_{RS} , & H_{RR} \end{bmatrix} \begin{bmatrix} \Phi_S \\ \Phi_R \end{bmatrix} = \underset{\substack{\uparrow \\ \text{energy}}}{\omega} \begin{bmatrix} \Phi_S \\ \Phi_R \end{bmatrix} \begin{matrix} \leftarrow \text{"relevent" degrees of freedom} \\ \leftarrow \text{the "rest"} \end{matrix}$$

$$H_{RS}\,\Phi_S + H_{RR}\,\Phi_R = \omega\,\Phi_R$$

$$H_{RS}\,\Phi_S = (\omega - H_{RR})\,\Phi_R$$

$$(\omega - H_{RR})^{-1} H_{RS}\,\Phi_S = \Phi_R$$

$$H_S\,\Phi_S + H_{SR}\,\Phi_R = \omega\,\Phi_S$$

$$\Big[H_S + \underbrace{H_{SR}\,(\omega - H_{RR})^{-1} H_{RS}}_{\text{frequency dependent}}\Big]\,\Phi_S = \omega\,\Phi_S$$

frequency dependent correction to low energy H_S due to the "rest".

$$\underbrace{\big[H_S + \Sigma_S(\omega)\big]\,\Phi_S = \omega\,\Phi_S}$$

corrected equation for the low energy part only is always energy dependent.

If we concentrate on the single particle Green's function, it will also contain frequency dependent corrections to $\mathcal{G}^0$ if there are processes not included in $\mathcal{G}^0$.

$$\mathcal{G}^0 = \left(\omega + \frac{\nabla^2}{2m} - V_{ion}\right)^{-1}$$

What is missing?
- electron-electron interaction $\quad \Sigma_{ee}$
- electron-phonon interaction $\quad \Sigma_{ep}$ $\Big\}$ dynamic
- spin-orbit (is static)

We can write $\quad \mathcal{G} = (\omega - H)^{-1} = \begin{pmatrix} \omega - H_S & -H_{SR} \\ -H_{RS} & \omega - H_{RR} \end{pmatrix}^{-1}$

$$= \begin{pmatrix} \left(\omega - H_S - H_{SR}\frac{1}{\omega - H_{RR}}H_{RS}\right)^{-1} & \ldots \\ \ldots & \ldots \end{pmatrix}$$

$$\mathcal{G}_S = \Big(\omega - H_S - \underbrace{H_{SR}\frac{1}{\omega - H_{RR}}H_{RS}}_{\text{can be called } \Sigma_S(\omega)}\Big)^{-1}$$

Dyson Equation:

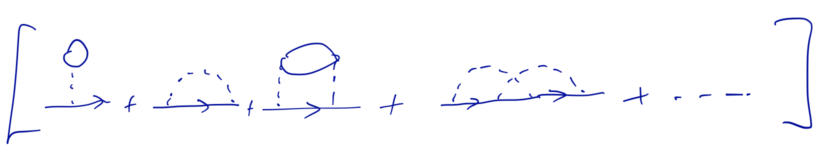
$$\mathcal{G}(\omega) = \left(\mathcal{G}_0^{-1} - \Sigma(\omega)\right)^{-1}$$

self energy describes interaction with the "rest"
- the real part describes the shifts and renormalization of the energy levels
- the imaginary part describes the lifetime of the quasiparticles.

In Feynman language:

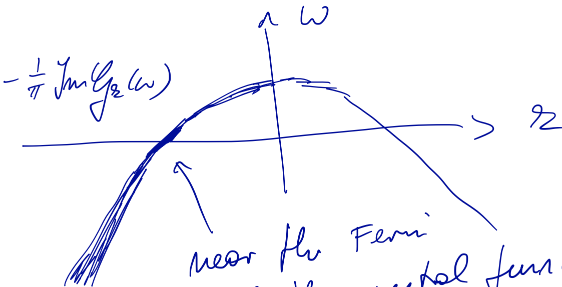
$$\mathcal{G}(\omega) = \mathcal{G}^0(\omega) + \mathcal{G}^0\,\Sigma\,\mathcal{G}^0 + \mathcal{G}^0\,\Sigma\,\mathcal{G}^0\,\Sigma\,\mathcal{G}^0 + \ldots$$

conclusion: Σ is the single particle irreducible part of $\mathcal{G}$: does not fall into two pices when cutting single $\mathcal{G}_0$



Now quasiparticles

$A_k(\omega) = -\frac{1}{\pi} \mathrm{Im}\, G_k(\omega)$



Note E_F corresponds $\omega = 0$.

near the Fermi level the spectral function in metals sharpen

follows from the Pauli ex. principle due to small phase space for the scattering

We want to describe the "effective" quasiparticle

$$G_k \approx \frac{1}{\omega - \varepsilon_k - \Sigma_k(\omega)}$$

because G is a matrix, and usually can not simply decouple bands

[Would require ε_k and $\Sigma_k(\omega)$ to be diagonal in the same basis which almost never happens]

near $\omega = 0$ we expand

$$\Sigma_k(\omega) = \Sigma_{k_F}(0) + \left(1 - \frac{1}{Z_k}\right)\omega + \frac{\partial \Sigma_k}{\partial k}(k - k_F) + \ldots$$

$$-i\left[\frac{\omega^2 + \pi^2 T^2}{\varepsilon^*}\right] + \ldots$$

fermi liquid result

$$1 - \frac{1}{Z_k} = \frac{\partial \Sigma_k}{\partial \omega} \quad \text{or} \quad Z_k = \left(1 - \frac{\partial \Sigma_k}{\partial \omega}\right)^{-1}_{\substack{\omega=0 \\ k=k_F}}$$

Then
$$G_k(\omega) \approx \frac{1}{\frac{\omega}{Z_k} - \varepsilon_k - \Sigma_k(\omega=0)} = \frac{Z_k}{\omega - \underbrace{Z_k(\varepsilon_k + \Sigma_k(\omega=0))}_{\text{"effective dispersion"}}}$$

weight is reduced i.e., quasiparticle

$$\varepsilon_k^{\mathrm{eff}} = Z_k\left(\varepsilon_k + \Sigma_k(\omega=0)\right)$$

Z_k makes it narrower because $Z_k < 1$. $\Sigma_k(\omega=0)$: shift

Mass of the dispersion:

$$\varepsilon_k + \Sigma_k(\omega=0) \approx \left(\left.\frac{d\varepsilon_k}{dk}\right|_{k=k_F} + \left.\frac{d\Sigma_k(\omega=0)}{dk}\right|_{k=k_F}\right)(k - k_F) + \ldots$$

Then
$$G_k(\omega) \approx \frac{Z_k}{\omega - (k-k_F)\, Z_k\left(v_F + \frac{d\Sigma_k}{dk}\right)} = \frac{Z_k}{\omega - \frac{(k-k_F)\, v_F}{m^*/m_b}}$$

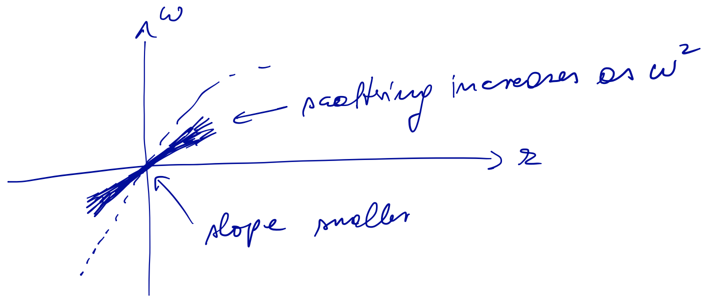
$$\frac{m^*}{m_b} = \frac{1}{Z_k}\left(1 + \frac{1}{v_F}\left.\frac{\partial \Sigma_k}{\partial k}\right|_{\substack{\omega=0 \\ k=k_F}}\right)^{-1}$$

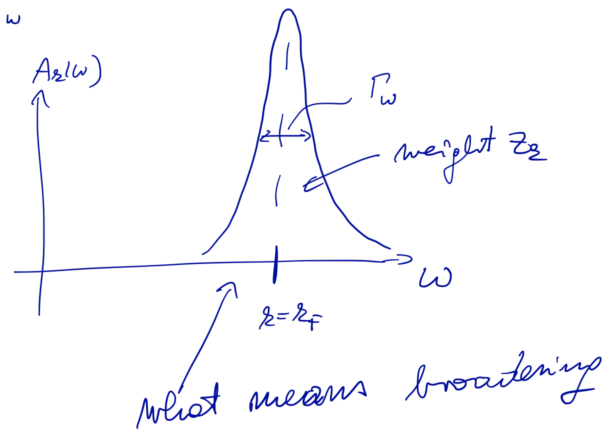
$\frac{1}{Z_k}$: typically the most important increases mass.

heavy fermions ($\sim 100 - 1000$)

We thus have
$$G_k(\omega) \approx \frac{Z_k}{\omega - (k-k_F)v_F\, m_b/m^* + i\underbrace{Z_k\left(\frac{\omega^2+\pi^2T^2}{\varepsilon^*}\right)}_{\Gamma_\omega}}$$

$$A_k(\omega) \approx \frac{1}{\pi}\,\frac{Z_k\,\Gamma_\omega}{\left(\omega - (k-k_F)v_F\, m_b/m^*\right)^2 + \Gamma_\omega^{\,2}}$$





$$G^0_k(t) = -i\, e^{-i\varepsilon_k t}$$

then
$$G_k(t) = -i\, Z_k\, e^{-i(\varepsilon_k - i\Gamma)t} = -i\, Z_k\, e^{-i\varepsilon_k - \Gamma_\omega t}$$

finite lifetime of the quasiparticle